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I.

DIMENSIONS OF THE FIXED STARS, WITH ESPECIAL REFERENCE TO BINARIES AND VARIABLES OF THE ALGOL TYPE.

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SINCE direct measurements cannot at present be made of the disks of the fixed stars, any information with regard to their dimensions derived from the amount and character of their light will have a value. This is the course ordinarily taken in the case of a satellite or small planet, and appears to deserve a more extended trial beyond the limits of the solar system than it has yet received. The principal objection to this method is the uncertainty in the numerical value of the intrinsic brightness or other constants involved, which cannot at present be measured with accuracy. The exact formulæ will therefore be first given, and hypotheses then introduced regarding the values of these constants.

Let B, b = the diameters of the Sun and of any given star, as seen from the Earth, expressed in seconds of arc.

Let l = the intrinsic brightness of the star, that of the sun being taken as unity; in other words, let l denote the ratio borne by the quantity of light emitted by the star to that emitted by the Sun from the same superficial area.

Let S, s = the light of the Sun and of the star expressed in stellar magnitudes by means of the scale of Pogson, in which a difference of one magnitude corresponds to the logarithmic ratio, 0.4. This ratio, expressed in numbers, is approximately 2.512.

Let p = the parallax of the star in seconds of arc.

The observed light of the star will be to that of the Sun as $l\,b^2$ is to B^2; the difference in their stellar magnitudes, or

$$s - S = 2.5 \log \frac{B^2}{l\,b^2} = 5 \log B - 5 \log b - 2.5 \log l.$$

Hence, $$\log b = \log B + 0.2\,S - 0.2\,s - 0.5 \log l.$$

The radius of the Sun equals 16′ 2″, and accordingly $B = 1924''$. The value of S is more uncertain. Various determinations of the ratio of the light of the Sun to that of Sirius have been made by different observers. In 1698, Huyghens found the value 756,000,000 by reducing the light of the Sun by a minute hole.* Wollaston, in 1829, compared the image of the Sun and of a lamp reflected in a silvered bulb of glass, and deduced the ratio 20,000,000,000.† Steinheil, in 1836, using the Moon as an intermediate standard of comparison, gave the value 3,840,000,000.‡ In 1861, Bond determined the relative light of the Sun and Moon by comparing their reflections in a glass globe with that of a Bengola light. Combining his measures with the comparisons of the Moon and Sirius by Herschel and Seidel, he deduced the value 5,970,500,000.§ In 1863, Clark found that, if the Sun was removed to 1,200,000 times its present distance and Sirius to 20 times its distance, they would appear equally bright, and equal to a sixth-magnitude star. Their ratio, consequently, equals 3,600,000,000.‖ Reducing these measures to magnitudes, we obtain the values, Huyghens, 22.20; Wollaston, 25.75; Steinheil, 23.96; Bond, 24.44; and Clark, 23.89. The mean of all of these is 24.05, with an average deviation of 0.84. The last three agree well, and give 24.10, with an average deviation of 0.23. Probably 24.0 is not far from the truth, and may be assumed to represent this ratio as closely as it is at present known. If we adopt —1.5 for the magnitude of Sirius, from the measures of Herschel and Seidel, we obtain for the stellar magnitude of the Sun —25.5.

* Cosmotheoros, La Haye, 1698.

† Phil. Trans., cxix. 28.

‡ Elemente der Helligkeits-Messungen, Munich, p. 24.

§ Mem. Amer. Acad., viii. N. S., p. 298.

‖ Amer. Jour. Sci., xxxvi. 76.

Substituting in the formula for log b given above, $B = 1924''$ and $S = -25.5$, we obtain $\log b = 3.284 - 5.100 - 0.2\,s - 0.5 \log l = 8.184 - 0.2\,s - 0.5 \log l$. This formula is exact, and would give the true diameter of any star if l was known.

An approximate value of l might be determined by the following method. Suppose that an electric current be passed through a platinum-iridium wire heating it to incandescence, and that the brightness of a short portion of it be compared with an artificial star when the current is varied by a known amount. As the current increases, the color of the light changes, the amount of the blue light increasing more rapidly than that of the red. The ratio of the two may be determined by inserting a double-image prism in the collimator of a spectroscope and viewing the wire through it. The two images may be made to overlap by any desired amount by varying the distance of the double-image prism from the slit of the collimator. The blue rays may thus be combined with the red, yellow, or green, as desired. The relative brightness of the two images may be varied by a Nicol placed in the eyepiece and turned through a known angle. We may thus combine any portion of the spectrum with any other part in such a proportion as to produce a tint to which the eye is especially sensitive. From the readings of the Nicol when different currents are passed through the wire, we may determine the varying proportion of any two rays, as the red and blue, when the wire is emitting a given amount of light. Observing in the same way the spectra of the Sun and star, and applying to them the law deduced from the observations of the wire, we obtain an approximate value of the comparative light emitted by equal areas of the two bodies. This will not be exact, since the effect of absorption is not allowed for, a difference of temperature being assumed to be the only cause of the observed difference in color. Probably the error will not be large, except perhaps in the case of the red stars. Until these measurements are made, we can do no better than to assume that $l = 1$, or that the emissive power is the same for the Sun and star. As a large portion of the stars have nearly the same color as the Sun, and a similar spectrum, this assumption will probably not be far from the truth. The term equivalent diameters may be conveniently applied to the quantities thus computed. They may be defined as the diameters the Sun would have if removed successively to such distances that it would equal in light stars of the given magnitudes. The expressions, equivalent densities and equivalent masses, will be used in the same manner to denote the densities or masses of bodies in their other properties resembling the Sun.

Table I. gives the equivalent diameters of stars of various magnitudes, assuming $l = 1$.

TABLE I. — EQUIVALENT DIAMETERS OF STARS OF VARIOUS MAGNITUDES.

Magn.	Diam.	Magn.	Diam.	Magn.	Diam.
0	0″.01528	5	0″.00153	10	0″.00015
1	.00964	6	.00096	11	.00010
2	.00608	7	.00061	12	.00006
3	.00384	8	.00038	13	.00004
4	.00242	9	.00024	14	.00002

The diameters corresponding to the intermediate magnitudes may be found from Table II., which gives the diameters for every tenth of a magnitude from 0.0 to 4.9.

TABLE II. — EQUIVALENT DIAMETERS OF STARS FOR EACH TENTH OF A MAGNITUDE.

Magn.	Diam.	Magn.	Diam.	Magn.	Diam.	Magn.	Diam.	Magn.	Diam.
0.0	0″.01528	1.0	0″.00964	2 0	0″.00608	3.0	0″.00384	4.0	0″.00242
0.1	.01459	1.1	.00920	2.1	.00581	3.1	.00366	4.1	.00231
0.2	.01393	1.2	.00879	2.2	.00555	3.2	.00350	4.2	.00221
0.3	.01330	1.3	.00840	2.3	.00530	3.3	.00334	4.3	.00211
0.4	.01271	1.4	.00802	2.4	.00506	3.4	.00319	4.4	.00201
0.5	.01213	1.5	.00766	2 5	.00483	3.5	.00305	4.5	.00192
0.6	.01159	1.6	.00731	2.6	.00461	3.6	.00291	4 6	.00184
0.7	.01107	1.7	.00698	2.7	.00441	3.7	.00278	4.7	.00175
0.8	.01057	1.8	.00667	2.8	.00421	3.8	.00266	4.8	.00168
0.9	.01009	1.9	.00637	2.9	.00402	3.9	.00254	4 9	.00160

When the magnitude is increased by five, the diameter will be reduced ten times, and the decimal point should accordingly be moved one place to the left. Thus, if a star of the 3.5 magnitude has a diameter of 0″.003, one of the 8.5 magnitude will have a diameter of 0″.0003 and one of the 13.5 magnitude, 0″.00003. The diameter of Sirius would be that corresponding to —1.5 magnitudes, or 0″.03, were it not that l is probably greater than 1 owing to the blue color of the star, and the diameter consequently less.

Should future measurements render some other value of S more probable, Tables I. and II. can still be used, merely changing s by the same amount that S is altered.

The smallest star that can be seen in the 15-inch telescope of the Harvard College Observatory has a magnitude of about 15.5, and a corresponding equivalent diameter of 0.″000012.

When the parallax of a star is known, these principles may be applied to determining its linear diameter. If the Sun was removed to the distance of the star its diameter would have the same ratio to the parallax that the chord of the Sun's diameter, as seen from the Earth, has to unity. It would therefore equal

$$2p \sin 16' \, 2'' = 0.00933 p.$$

Table III. gives the light in stellar magnitudes which would be emitted by the Sun if removed to such a distance that its parallax would have the value given in the first column.

TABLE III. — PARALLAX.

Par.	Magn.	Par.	Magn.
0."1	6.07	0."6	2.18
0.2	4.57	0.7	1.84
0.3	3.68	0.8	1.56
0.4	3.06	0.9	1.30
0.5	2.58	1.0	1.07

If the parallax of *α Centauri* is assumed to be 0″.9, the Sun as seen from it will appear as a star of the 1.3 magnitude. The light of *α Centauri* is not known with much certainty, as we have to depend upon eye estimates. Assuming the magnitude of the two components to equal 0.0 and 3.0, we find that if $l = 1$ for both of them, their diameters will be 1.82 and 0.46 times that of the Sun. The parallax of 61 *Cygni* may in like manner be assumed to be 0″.3, and the magnitude of its components 5.0 and 6.0. The Sun would then appear, from this distance, as a star of the 3.7 magnitude, and the diameter of the two components, compared with that of the Sun, if their emissive powers are the same, will be 0.55 and 0.35.

I. Binary Stars.

In the case of a binary star, another equation of condition may be introduced from Kepler's third law. Let N denote the mass of the binary in terms of that of the Sun, P the period of revolution in years, a the semi-axis major, or mean distance of the components, and b the equivalent diameter, or the diameter of a star having the same mass as the binary, and the same density and intrinsic brightness as the Sun.

Comparing the binary with the system formed by the Sun and Earth seen at the same distance, we see that the two systems have masses in

the ratio of N to 1, mean distances in the proportion of a to p, and periods of revolution as P to 1. Accordingly, by Kepler's law, $N : 1 = \frac{a^3}{P^2} : \frac{p^3}{1}$, or $N = \frac{a^3}{p^3 P^2}$. But $N = \frac{b^3}{(0.00933\,p)^3}$, since $0.00933\,p$ will equal the diameter of the Sun at the distance of the binary. Hence, equating these two values of N, p is eliminated, and we have $b = 0.00933\ a\ P^{-\frac{2}{3}}$. The stellar magnitude corresponding to the diameter, b, may now be found from Tables I. and II. So far, no hypothesis has been introduced, and the errors in these quantities will depend only on the errors in the photometric measurements and in the micrometric determination of the elements of the orbit.

If now we could find the value of l for each of the components, as suggested above, we could determine the true diameter of the two stars, and from their orbits, and the mass of the binary, deduce their average densities. Until these measures are made, we can do no better than assume that both stars have the same density, and that $l = 1$ for each. On this hypothesis, if b_1, b_2 are the equivalent diameters of the two components, and b the equivalent diameter of the binary as computed from the time of revolution and mean distance, the density will equal $\frac{b^3}{b_1^3 + b_2^3}$.

Since the value of the parallax is eliminated, it follows that these considerations will not aid the determination of the distance of a binary. The time of revolution of a binary would remain unchanged if removed to double the distance, provided that the linear distance of the components and their diameters were increased in the same proportion, or that the angular dimensions of the system remained unchanged. In other words, the observed time of revolution of a binary system is wholly independent of its distance from the observer.

The relative masses of the two components could be determined micrometrically and independently of the above methods, by measuring the position of each component from the adjacent stars. If this was repeated at intervals during an entire revolution of the binary, the components would be found to have described similar ellipses whose dimensions would be inversely proportioned to the masses. From the *Proc. Roy. Astron. Soc.*, xl. 235, it would appear that Mr. Gill will apply this test to the components of *α Centauri*. If the difference in light is three magnitudes, and the intrinsic brightness and densities the same for the two components, the ratio of the masses would be as 63 to 1. The semi-axis major of the ellipse described by the larger star would therefore be, according to the elements given by Hind,

$\frac{21.80}{64} = 0''.34$. Owing to the inclination of the plane of the orbit the apparent ellipse would be much less than this. Some other stars would appear better adapted to this test. The smaller difference in light more than compensates for the smaller orbit. From the data given in Table V., the semi-axis major of the ellipse described by several stars has been computed. The name of the star is followed by the time of revolution in years, and the semi-axis of the ellipse described by the larger component; γ *Coronæ Australis*, 45, $1''.2$; ξ *Ursæ Majoris*, 60, $0''.8$; 70 *Ophiuchi*, 94, $0''.4$; ξ *Boötis*, 127, $0''.2$; γ *Virginis*, 185, $2''.0$. Some others might give a larger apparent orbit, but a very long time would be required to detect the motion. When the inclination of the orbit is not zero, the apparent ellipse will be less than that computed in this manner in the same proportion that the apparent orbit is less than the real orbit described by the companion. Similar observations might be made on any double stars whose components appear to be physically connected. The proper motion, however, complicates the phenomenon, and cannot be distinguished from the orbital motion as long as the latter appears to be rectilinear.

So many large telescopes are now devoted to the measurement of double stars that there is great danger of an unnecessary duplication of work. A valuable contribution might be made to our knowledge of stellar motion by determining the positions of the components of a double star with regard to several adjacent stars. Even if the masses of the components could not thus be determined, we should at least provide the material for an accurate measurement of their proper motions in the future. The same may be said of the determination of the proper motions of other stars, which could be observed in this way with much greater precision than by the usual meridian observations. Useful work could be done by an observer unprovided with means for measurements by simply examining a large number of double stars and stars having a large proper motion, and noting the approximate position and distances of any adjacent stars near enough and bright enough for accurate measurement. A list would thus be formed from which the selection of suitable objects would be easy.

The spectroscope, which has opened so rich a field for work in astronomy, may be applied also to the study of the binary stars. If measurements could be obtained of the approach or recession of the two components, several interesting conclusions could be derived from them. A single measurement would not give the relative masses of the components, since the effect of the proper motion cannot be dis-

tinguished from that caused by the inequality of the masses. The proper motion may be eliminated if the observations are repeated in different parts of the orbit of the binary, since its effect would be always the same, while that due to the inequality of the masses would be continually altering, becoming zero and altering its sign twice during each revolution. If the ratio of the masses could be determined micrometrically as described above, the measures with the spectroscope would determine the component of the proper motion in the direction of the line of sight. The principal use of the measures with the spectroscope would be to determine the true dimensions of the orbit, and consequently the distance of the binary.

Let ☊ denote the position angle of the node of the binary, i the inclination of the plane of its true to that of its apparent orbit, s the distance, and p the position angle at the time of observation; let ds and dp represent the annual changes in these quantities. Let us make a transformation to a system of rectangular co-ordinates in which the axis of X shall coincide with the line of nodes, the axis of Z coincide with the line of sight, and the axis of Y be perpendicular to both of them. Then dz will equal the annual change in the distances of the two components from the observer, or will measure in seconds of arc the same quantity that the spectroscope measures by the difference in velocity of the two components. But

$$dz = dy \tan i \text{ and } y = s \sin (p - \Omega);$$

hence $$dz = \tan i \sin (p - \Omega)\, ds + s \tan i \cos (p - \Omega)\, dp.$$

Substituting the proper numerical values we obtain dz in seconds of arc; it should be remembered that dp must be expressed in terms of the radius, or $57^\circ.3$ must be taken as the unit. This method may be employed if we have an ephemeris of the star, the inclination of the orbit, and the position angle of the line of nodes. If the elements of the orbit are given without an ephemeris, a different formula must be used. Let ρ denote the real distance of the components, and u the angle from the node measured in the plane of the orbit. If a system of co-ordinates is employed such that X' lies in the line of nodes, Y' perpendicular to it in the plane of the orbit, and Z' in the line of sight, we have

$$y' = \rho \sin u, \text{ and } dz' = dy' \sin i = \sin i \sin u \, d\rho + \rho \sin i \cos u \, du.$$

If the orbit is circular, u increases uniformly with the time, and ρ is constant and equals a; hence $dz' = a \sin i \cos u \, du$. If in this expression $du = \frac{2\pi}{P}$, or denotes the fraction of the orbit

traversed in one year, $dz' = \frac{2\pi a \sin i}{P} \cos u$. The maximum value of this expression occurs when $u = 0°$ or π, and is $\frac{2\pi a \sin i}{P}$. If the orbit is elliptical, ρ and u may be deduced from the elements, and dz may be expressed as a function of the eccentricity, node, and time, multiplied by the factor, which is constant for each orbit, $\frac{a \sin i}{P}$.

Let V denote the velocity of light, v the velocity of approach of a star, λ the wave-length of a given ray of light, and l the corresponding change it undergoes, due to the velocity. Then $V + v : V = \lambda + l : \lambda$ or $v = V \frac{l}{\lambda}$; v and V are commonly expressed in kilometers per second, l and λ in ten-millionths of a millimeter; $V = 300000$. The line F is frequently used in these measures, and for it $\lambda = 4865$. Substituting these values, $v = 62\,l$. For the D line, $\lambda = 5900$, and since the interval between the two components equals 6, a velocity of 305 kilometers per second will be required to produce a deviation equal to the interval between these lines. It will be more convenient to measure the velocity of a star in terms of m, the annual motion, taking the distance from the Earth to the Sun as a unit. This may then be reduced to seconds of arc, if the distance of the star is known, by multiplying by the parallax p. Light traverses the distance from the Earth to the Sun in about 498 seconds, or would traverse 63300 times this distance in a year. Accordingly, $v = 63300 \frac{l}{\lambda}$; for the F line $v = 13\,l$, for the interval of the D lines, $v = 64\,l$. If l is positive or the line moves toward the red end, it denotes that the star is receding from the observer. We have thus two values of the relative motion of the stars in the line of sight; one, $d\,z$, deduced by computation from the micrometer measurements; the other, $v\,p$, or $13\,l\,p$, if the F line is observed, found by the spectroscope. Equating these values, since p is the only unknown quantity, $p = \frac{d\,z}{13\,l}$. The dimensions of the orbit are now found directly, since $\frac{a}{p}$ will equal the semi-axis major in terms of the distance of the Sun from the Earth.

It not unfrequently happens that we have an estimate of the difference in magnitude of the two components of a double star by one observer using a telescope, and also an estimate of their combined light by another observer viewing them with the unassisted eye. From these data we wish to determine the brightness of either component alone. Sometimes we have the opposite problem, given the magnitude of the separate stars to find that of both, as seen by the eye or in a

telescope not capable of separating them. Let l denote the light of the fainter star in terms of the brighter, and m the magnitude of the fainter minus the magnitude of the brighter. Then, on Pogson's system, $m = -2.5 \log l$. If M is the magnitude of the brighter star minus that of a star equivalent to the two combined, or having the light $(1 + l)$, then $M = -2.5 \log (1 + l)$. From these formulæ we can always find the corresponding values of M and m. The maximum value of $M = 0.75$ when m is zero or the stars are equal. Table IV. enables us to determine M to the nearest tenth of a magnitude for any value of m. As an example, suppose two stars have magnitudes 2.0 and 3.0; then $m = 3.0 - 2.0 = 1.0$, and M, from the table, lies between 0.35 and 0.45 or equals 0.4. The light of both combined will therefore equal $2.0 - 0.4 = 1.6$.

TABLE IV. — COMBINATION OF TWO STARS.

M.	m.	m'.
0.05	3.32	1.90
0.15	2 07	1.06
0.25	1.47	0.64
0.35	1.05	0.34
0.45	0.72	0.11
0.55	0 45	—
0.65	0.22	—
0.75	0.00	—

It is sometimes convenient to know what would be the magnitude of a star whose mass was equal to that of the two components of a double star of the same density and brightness. Let m' equal the difference in magnitudes of the two components, and l and n, the light and mass of the fainter in terms of the brighter. Then

$$m' = -2.5 \log l = -2.5 \log n^{\frac{2}{3}} = -1.67 \log n,$$

since the light is proportional to the square, and the mass to the cube, of the diameter. If then M equals the magnitude of the brighter component minus that of both combined, we shall have $M = 1.67 \log (1 + n)$, from which M is determined as before from any given value of m'. The third column of Table IV. gives the value of m' corresponding to every odd twentieth of a magnitude of M. The value of the latter may thus always be determined to the nearest tenth of a magnitude. The maximum value of M is 0.50, when $m' = 0$. Adopting the same magnitudes as in the last example, if two stars have the magnitudes of 2.0 and 3.0, m' will equal 1.0. This value from the third column of Table IV. will correspond to a value of M lying between 0.15 and 0.25, or will equal 0.2. The magnitude

of a star having the same mass as the binary will therefore have a magnitude 2.0 — 0.2 = 1.8.

Most of the binary stars whose orbits have been computed are compared in Table V. The successive columns give a current number, the name of the star, the number of the Dorpat Catalogue, the right ascension and declination for 1880, the semi-axis major in seconds, the eccentricity, the period in years, and the inclination of the plane of the orbit in degrees. The next two columns give the magnitudes of the components as estimated by Struve. Three of the stars are not contained in the Dorpat Catalogue, and for them the magnitudes given have been assumed. The next column gives the equivalent diameter $0.00933\ a\ P^{-\frac{2}{3}}$, or the magnitude of a star having the mass of the binary and the density and brightness of the Sun. From the magnitudes of the components we may compute, by the third column of Table IV., the brightness of a star having the same mass as the binary and the same brightness and density as its components. Subtracting from this quantity that given in the preceding column gives the next column. If these quantities were small, we might assume that they were due to errors in the assumed magnitudes of the stars. Their variations are, however, far too large to be explained in this way. As they are almost all negative, we may infer that the assumed light of the Sun is too small, or that a larger value should have been given on page 2 to S. A great part of the difference must be ascribed to variations in the density or brightness of the stars. We have at present no way of discriminating between these causes. Such a method as has been proposed on page 3 for determining l would serve to distinguish them. Until then, it will be convenient to reduce this difference from magnitudes to the relative diameters of two stars of equal density and brightness, one having a mass, the other emitting a light equal to that of the binary. Assuming the diameter of the first of these stars as a unit, the diameter of the other is given in the next column, and may be denoted by C. In almost all cases this quantity is greater than unity, from which we should infer that most of the stars enumerated are either much brighter or much less dense than the Sun, unless, as suggested above, the measurements of the light of the Sun are largely in error. Let d denote the density, b the brightness of the components of the binary, and D the equivalent diameter of the binary in terms of the same unit as C. Then $D^2 : C^2 = 1 : b$, and $D^3 : 1^3 = 1 : d$; eliminating D, $C = \frac{b^{\frac{1}{2}}}{d^{\frac{1}{3}}}$, or the brightness is proportional to the square of C and the density inversely as its cube. If

TABLE V.—BINARY STARS.

No.	Name.	Σ.	R. A. 1880.		Dec. 1880.		a	e	P.	i	A.	B.	Equiv. Magn.	O.—C.	Relat Diam.	$\frac{a \sin i}{P}$	Computer.
			h.	m.	°	′	″			°						″	
1	42 Comæ Berenicis.....	1728	13	4.1	+ 18	10	0.66	0.48	25.7	90.0	6.0	6.0	6.7	— 1.2	1.74	0.026	O. Struve.
2	ζ Herculis............	2084	16	36.8	+ 31	49	1.36	0.41	34.6	51.1	3.0	6.5	5.5	— 2.5	3.16	0.031	Flammarion.
3	Anon. Leonis.......	3121	9	10.7	+ 29	7	0.71	0.26	37.0	74.2	7.5	7.8	7.0	+ 0.1	0.96	0.019	Doberck.
4	η Coronæ Borealis	1937	15	18.2	+ 30	43	0.99	0.29	40.2	60.4	5.2	5.7	6.4	— 1.5	1.99	0.021	Flammarion.
5	221 B Ophiuchi.........	2173	17	24.7	— 0	58	1.01	0.13	45.4	80.5	5.8	6.1	6.6	— 1.2	1.74	0.022	Dunér.
6	α Canis Majoris.......	....	6	39.7	— 16	32	7.11	0.61	49.4	47.1	— 1.5	8.5	2.5	— 4.0	6.31	0.106	Auwers.
7	γ Coronæ Australis ...	....	18	58.3	— 37	14	2.40	0.70	55.6	68.6	5.5	5.5	5.0	0.0	1.00	0.040	Schiaparelli.
8	ξ Ursæ Majoris........	1523	11	11.8	+ 32	13	2.58	0.38	60.7	56.3	4.4	5.3	5.0	— 0.8	1.44	0.037	Hind.
9	ζ Cancri............	1196	8	5.3	+ 18	1	0.91	0.35	62.4	20.7	5.0	5.7	7.2	— 2.4	3.02	0.005	O. Struve.
10	α Centauri	....	14	31.8	— 60	20	21.80	0.67	85.0	82.3	0.0	3.0	0.8	— 0.8	1.44	0.254	Hind.
11	"	....	"	"	"	"	18.45	0.53	88.5	79.4	"	"	1.2	— 1.2	1.74	0.205	Doberck.
12	70 Ophiuchi..........	2272	17	59.4	+ 2	33	4.88	0.39	92.8	62.1	4.0	6.3	4.2	— 0.2	1.10	0.046	Flammarion.
13	γ Coronæ Borealis	1967	15	37.7	+ 26	41	0.70	0.35	95.5	85.2	4.0	7.0	8.4	— 4.4	7.59	0.007	Doberck.
14	ξ Scorpii............	1998	15	57.8	— 11	2	1.26	0.08	95.9	68.7	4.9	5.2	7.2	— 2.7	3.47	0.012	"
15	Anon. Cassiopeiæ...	3062	23	59.9	+ 57	46	1.27	0.46	104.4	32.2	6.9	8.0	7.3	— 0.5	1.26	0.006	"
16	ω Leonis............	1356	9	22.0	+ 9	35	0.89	0.54	110.8	64.1	6.2	7.0	8.1	— 2.1	2.63	0.007	"
17	p Eridani...........	407	3	24.3	— 11	32	3.82	0.38	117.5	44.7	8.2	10.7	5.1	+ 3.1	0.24	0.023	"
18	25 Canum Venaticorum	1768	13	52.1	+ 36	54	0.75	0.66	124.5	51.5	5.7	7.6	8.7	— 3.1	4.17	0.005	"
19	ξ Boötis............	1888	14	45.8	+ 19	36	4.86	0.71	127.4	36.9	4.6	6.9	4.7	— 0.1	1.05	0.023	"
20	γ Virginis	1670	12	35.6	— 0	47	3.39	0.87	175.0	0.0	3.0	3.0	5.9	— 3.4	4.79	0.000	Flammarion.
21	"	"	"	"	"	"	3.97	0.90	185.0	35.1	"	"	5.6	— 3.1	4.17	0.012	Thiele.
22	τ Ophiuchi...........	2262	17	56.5	— 8	11	1.40	0.61	217.9	58.7	5.0	5.7	8.1	— 3.3	4.57	0.005	Doberck.
23	η Cassiopeiæ..........	60	0	41.8	+ 57	11	9.83	0.58	222.4	53.5	4.0	7.6	3.9	+ 0.1	0.96	0.036	"
24	λ Ophiuchi..........	2055	16	24.9	+ 2	15	1.19	0.49	233.9	44.7	4.0	6.1	8.6	— 4.6	8.32	0.004	"
25	44 Boötis............	1909	14	59.8	+ 48	7	3.09	0.71	261.1	70.1	5.2	6.2	6.7	— 1.7	2.19	0.011	"
26	μ^2 Boötis............	1938	15	20.0	+ 37	46	1.47	0.60	280.3	40.0	6.7	7.3	8.4	— 2.0	3.51	0.003	"
27	36 Andromedæ	73	0	48.0	+ 22	58	1.54	0.65	349.1	41.6	6.2	6.8	8.6	— 2.7	3.47	0.003	"
28	γ Leonis............	1424	10	13.4	+ 20	27	1.98	0.73	407.0	43.1	2.2	3.6	8.3	— 6.2	17.38	0.003	"
29	δ Cygni	2579	19	41.2	+ 44	50	2.31	0.29	415.1	37.7	3.0	7.9	8.0	— 5.0	10.00	0.003	Behrmann.
30	61 Cygni	2758	21	1.4	+ 38	7	15.40	..	452.0	..	5.1	6.0	4.0	+ 0.9	0.66	—	——
31	σ Coronæ Borealis	2032	16	10.2	+ 34	10	5.88	0.75	845.9	31.9	5.1	6.4	7.0	— 2.0	2.51	0.004	Doberck.
32	α Geminorum.........	1110	7	27.0	+ 32	9	7.43	0.33	1001.2	44.6	2.7	3.7	6.7	— 4.2	6.92	0.005	"
33	ζ Aquarii............	2909	22	22.6	— 0	38	7.64	0.65	1578.3	44.7	4.0	4.3	7.3	— 3.7	5.50	0.003	"

then the star has the same density as the Sun, the square of C will give its brightness. Again, if the star has the same brightness as the Sun, its density will equal one divided by the cube of C.

The product of the semi-axis major by the sine of the inclination and divided by the period is given in the last column but one. It serves as a measure of the annual approach or recession of the two components. Neglecting the eccentricity, the maximum motion in seconds will equal this quantity multiplied by $2\,\pi = 6.28$.

The last column gives the name of the astronomer by whom the orbit was computed, which is adopted in this discussion.

An inspection of the last column but one shows that the value of $\frac{a \sin i}{P}$ in several cases amounts to $0''.03$ or even more. Neglecting the eccentricity, the maximum motion would therefore equal $2\,\pi$ times this quantity, or nearly $0''.2$. The eccentricity in some cases would diminish the motion, but in other cases it would increase it. An eccentricity of 0.5 might vary it from $0''.1$ to $0''.4$, according to the position of the peri-astron. This value of $\frac{a \sin i}{P}$ would probably be even larger for some of the recently discovered stars, in which P is still smaller than in the stars given in the table. It is commonly supposed that the parallax of an average first-magnitude star does not much exceed $0''.1$. That of a sixth-magnitude star would then be about $0''.01$ unless the fainter stars are really smaller than the brighter, or unless there is a perceptible absorption of light in space. Substituting the values $d\,z = 0''.2$, $p = 0''.01$, in the formula for the F line, $p = \frac{d\,z}{13\,l}$, given on page 9, we deduce $l = \frac{d\,z}{13\,p} = 1.5$. Accordingly the difference in the positions of the F line would be 1.5 times as great as the deviation observed in the case of Sirius. As the spectra of the two components could be observed in turn (or perhaps simultaneously) without disturbing the spectroscope, many of the causes of uncertainty present in similar measures of single stars would be removed.

In any case, if the F line could be seen in both components, we could assign a limit within which we could be certain that it was the same for both, and this would give a value of the parallax which must be less than the true parallax. A determination of the outside limit of distance of a star would appear to have nearly the same importance as the inside limit of distance found by micrometric distance. Moreover it does not seem probable that a star will be found whose parallax is very large, or previous observation might have detected it.

The search for a star with a very small parallax seems more hopeful, since it could not have been detected by other measures.

The observation would have value if we could determine the direction of the motion, even if we could not measure its amount, since it would show which portion of the orbit was turned towards the observer. This cannot be found from the micrometric measurements, since, although we can obtain from them the amount of the inclination, we cannot determine its sign.

It is also possible that some method of greater delicacy may be discovered, so that the spectroscope may be replaced by a more sensitive instrument, as it has been by the interferential refractometer in measuring the index of refraction of gases.

The semi-axes major of Σ 3121, 1768, 2262, and 2055 are not given in the original publications of the orbits. The values inserted in Table V. are those given in the Handbook of Double Stars, by Messrs. Crossley, Gledhill, and Wilson. This work has also proved most useful in various ways in the preparation of this paper. The value of a given by Dr. Auwers for *α Canis Majoris* is 2.33. This relates to the ellipse described by the bright star. As the companion is assumed to have a mass $\frac{1}{2.05}$ times as great as this, the value of a must be multiplied by 3.05, and therefore $2.33 \times 3.05 = 7.11$ is the value adopted. It is obvious that for this star the intrinsic brightness of the two components is by no means the same. If the density is the same, the diameter of the companion would be 0.79 that of the primary. The area of its disk would be 0.62, while its light* is only 0.0001 of that of its primary. The very large relative diameter of *γ Leonis* is remarkable. Its brightness must be about three hundred times that of the Sun, if its density is the same. On the other hand, if no brighter than the Sun, its density would be only one seventh of that of atmospheric air at the standard density and pressure, to give it a sufficient bulk to emit its observed light. If the other binaries have the same density as the Sun, their brightness must vary from 100 in the case of *δ Cygni* to 0.06 in the case of *p Eridani*, the brightness of the Sun being taken as the unit. The semi-axis major and period of 61 *Cygni* are taken from Newcomb and Holden's Astronomy. Although this star is commonly regarded as a binary, the evidence in favor of this view seems to depend upon the large proper motion of both components, and the fact that both appear to be comparatively near the Sun. It is doubt-

* Ann. Harv. Coll. Observ., xi. 177.

ful whether the observations yet made are sufficiently exact to prove a connection between the components. To establish this proposition, and also as an example of a convenient means of distinguishing a binary star from one which is optically double, the following investigation is given of the more important observations of 61 *Cygni*. We cannot conclude that a star is binary unless the path described by one of its components appears to be concave with respect to the other. If the motion appears to be rectilinear, it is more probably that due to the proper motion of one of them, or rather to the combined effect of the proper motions of both. On the other hand, if the path is convex, it is extremely probable that there is a real connection between the two, as there is no instance known of a star describing a curved path due to proper motion alone. The motion, if rectilinear, should also be uniform, while, if curved, the motion should be most rapid when nearest the other star. The law that the area described by the radius vector is proportional to the time, cannot be used to distinguish between those motions, since it will apply to both.

Suppose that the measures are transformed to a system of rectangular co-ordinates, having one component as the origin, and the axis of X nearly parallel to the path of the other component. Except for the accidental errors, the value of y, if the motion is rectilinear, should be the same for all the observations from the beginning to the end of the series. If the axis of X is not exactly parallel to the line of motion the values of y should increase slowly from one end of the series to the other. If they are corrected by an amount which will be proportional to the time, this variation should disappear. If the star is binary, however, the value of y will vary, in general having its greatest value during the middle of the period, and being smaller at the beginning and end.

The values of x, if the motion is rectilinear, will vary uniformly with the time, and, if corrected by a constant, plus another constant multiplied by the time, will leave residuals that are very small. If the motion is curved, on the other hand, this condition will not be fulfilled.

A reduction of the observations of 61 *Cygni* is given in Table VI. Of the measurements made during the last half-century only those made by the Struves and by Dembowski have been employed. The position angles are first corrected for precession and reduced to the epoch of 1880 by the formula

$$0^\circ.00557 \sin \alpha \sec \delta \, (t - 1880) = -0^\circ.005 \, (t - 1880).$$

A simple computation shows that the direction of the motion is nearly that of the position angle 256°. We have accordingly, $y = s \cos (p - 166°)$, and $x = s \sin (p - 166°)$. In the successive

TABLE VI. — PATH OF 61 CYGNI.

No.	Date.	Obs.	Cor. p.	s.	x	y	Δx	Δy
1	1753.8	Bradley.	34°.8	19.63	—12.93	14.77	—0.29	—0.43
2	1778.0	Mayer.	50.4	15.24	— 6.59	13.74	+0.97	—1.46
3	1781.9	Herschel.	53.3	16.33	— 6.30	15.07	+0.44	—0.13
4	1793.6	Lalande.	52.3	14.87	— 5.98	13.61	—1.70	—1.59
5	1800.0	Piazzi.	69.8	19.27	— 2.08	19.16	+0.86	+3.96
6	1805.0	"	78.1	14.50	+ 0.53	14.49	+2.42	—0.71
7	1812.3	Bessel.	78.8	16.74	+ 0.82	16.72	+1.18	+1.52
8	1813.8	Lindenau.	68.8	16.56	— 2.08	16.43	—2.04	+1.23
9	1814.5	W. Struve.	68.6	15.20	— 1.96	15.08	—2.06	—0.12
10	1820.5	"	83.2	15.11	+ 1.89	14.99	+0.53	—0.21
11	1822.7	"	85.5	14.93	+ 2.46	14.73	+0.63	—0.47
12	1828.7	"	89.1	15.31	+ 3.47	14.91	+0.38	—0.29
13	1831.7	"	90.9	15.63	+ 4.02	15.11	+0.30	—0.09
14	1832.8	"	91.8	15.79	+ 4.30	15.20	+0.35	0.00
15	1835.6	"	93.6	15.97	+ 4.83	15.22	+0.29	+0.02
16	1836.6	"	94.2	16.08	+ 5.02	15.27	+0.37	+0.07
17	1837.7	"	95.2	15.93	+ 5.24	15.04	+0.26	—0.16
18	1843.5	O. Struve.	98.8	16.67	+ 6.46	15.44	+0.26	+0.24
19	1847.5	"	100.7	17.02	+ 7.11	15.46	+0.07	+0.26
20	1850.3	"	102.3	17.18	+ 7.61	15.40	+0.01	+0.20
21	1851.8	"	103.5	17.34	+ 8.02	15.38	+0.08	+0.18
22	1852.7	"	104.4	17.46	+ 8.30	15.36	+0.17	+0.16
23	1854.2	"	105.1	17.57	+ 8.54	15.35	+0.10	+0.15
24	1857.2	"	106.4	18.02	+ 9.12	15.55	+0.05	+0.35
25	1860.8	"	108.6	18.22	+ 9.82	15.35	—0.01	+0.15
26	1868.5	"	112.4	18.81	+11.16	15.14	—0.28	—0.06
27	1874.7	"	116.1	19.42	+12.51	14.85	—0.24	—0.35
28	1854.7	Dembowski.	105.4	17.29	+ 8.49	15.06	—0.06	—0.14
29	1855.8	"	106.0	17.34	+ 8.67	15.02	—0.11	—0.18
30	1856.6	"	106.3	17.45	+ 8.80	15.07	—0.15	—0.13
31	1857.6	"	107.2	17.73	+ 9.19	15.17	+0.03	—0.03
32	1858.5	"	107.7	17.73	+ 9.32	15.08	—0.02	—0.12
33	1862.8	"	109.3	18.36	+10.08	15.35	—0.17	+0.15
34	1863.4	"	109.5	18.37	+10.14	15.32	—0.23	+0.12
35	1864.7	"	110.3	18.53	+10.44	15.31	—0.21	+0.11
36	1865.6	"	110.8	18.57	+10.60	15.25	—0.24	—0.05
37	1867.2	"	111.6	18.72	+10.90	15.22	—0.27	+0.02
38	1868.7	"	112.7	18.83	+11.25	15.10	—0.24	—0.10
39	1869.7	"	113.4	18.96	+11.52	15.06	—0.18	—0.14
40	1870.6	"	113.9	19.16	+11.77	15.12	—0.12	—0.08
41	1871.6	"	114.2	19.23	+11.89	15.10	—0.21	—0.10
42	1872.6	"	114.3	19.33	+11.98	15.11	—0.33	—0.09
43	1873.6	"	114.8	19.44	+12.18	15.15	—0.34	—0.05
44	1874.5	"	115.3	19.50	+12.35	15.05	—0.35	—0.15
45	1875.6	"	115.9	19.58	+12.56	15.02	—0.38	—0.18

columns of Table VI. are given a current number, the date, the name of the observer, the corrected position angle, the distance, and

the values of x and y. The value of y is approximately $15''.20$; that of x, $0''.21\ (t - 1814)$. Residuals are accordingly given in the last two columns by subtracting the values of y and x thus obtained from those observed.

The residuals in the last two columns are evidently not due to accidental errors, but whether they are caused by curvature of the path or systematic errors of the observer is less evident. The first nine sets are so discordant, that little dependence can be placed upon them. The values of Δy show a very slight increase, followed by a diminution in the later values. Δx seems to diminish slowly, the later values of the Struves and of Dembowski being somewhat less than the earlier. The curvature is so slight, that it has been thought to indicate an hyperbolic orbit. The observations so far made will however be very nearly satisfied by a large circular orbit seen obliquely, so that the part described during the last century has been that near the end of the minor axis of the apparent ellipse.

If we take the mean of the residuals, we find the values for Δx of $0''.25$ and for Δy of $0''.15$. As these include all kinds of systematic errors, the deviations from a straight line can scarcely be regarded as certain.

II. Variable Stars of the Algol Type.

Variable stars may be divided into several classes, according to the nature of the fluctuations of their light. First, temporary stars, which appear suddenly, and gradually fade away during the next few months. The most famous star of this class is that observed in 1572, by Tycho Brahe. The new stars in *Corona Borealis* in 1866 and in *Cygnus* in 1876, are recent examples of this class. Second, a large part of the variable stars pass from their maximum to their minimum and back again, in from six months to two years, the period and the brightness at the maximum and minimum being somewhat variable. The change in light is generally very great, amounting to several hundred, or even thousand times. The most striking examples of this class are *o Ceti* and χ *Cygni*. Thirdly, we have the slight changes to which many (or, according to Dr. Gould, most) stars are liable. These changes seem to be irregular in many cases; at least, their law is not yet known. Examples of this class are furnished in α *Orionis* and α *Cassiopeiæ*. Fourthly, certain stars continually vary, going through a series of changes in the course of a few days, which appears to be repeated exactly. Two causes seem here to be superimposed, one producing one

maximum and one minimum in each period, the other two maxima and two minima in the same time. As examples, β *Lyræ* and δ *Cephei* may be noted. Fifthly, we have a class of stars which during the greater part of the time remain unchanged in brightness, but at regular intervals lose in the course of a few hours a large part of their light, and regain it with equal rapidity. These changes appear to be repeated with the greatest regularity, so that the interval can be computed in some cases within a fraction of a second. *Algol*, or β *Persei*, is the most striking example of this class to which δ *Cancri* and δ *Libræ* also belong.

Various theories have been advanced to account for these phenomena. Probably different causes act in the case of the different classes. One theory would assume that by a collision, or by the liberation and ignition of a vast amount of hydrogen, the star was suddenly heated to incandescence, and gradually lost its light by cooling. This explanation would apply only to stars of the first class; it is strengthened in the case of the new star in *Cygnus* by the observations with the spectroscope. The spectrum gave at first the lines of incandescent hydrogen which disappeared as the light faded. It has been urged that, to account for the rapid cooling, the star must have been small, perhaps only a few miles in diameter, and consequently not very distant. This view is contradicted by the absence of perceptible parallax. If we consider how quickly a meteorite becomes heated, and again gives up its heat, this argument loses its force. The star may be large and distant, the surface only being heated, and soon losing its heat by radiation and conduction. This explanation appears more probable than that the light is cut off by clouds of smoke or steam, as has been suggested by some astronomers.

Stars constituted like our Sun, but in which the variations in size of the spots would be far greater, might undergo considerable changes in light. While it is difficult to account for the great changes in class two in this way, those in class three may be thus explained. A popular theory for the variation of stars of short period is that it is due to the revolution of the star upon its axis, when the different portions are of unequal brightness. The variation in light of Iapetus, the outer satellite of Saturn, is commonly explained in this way. A similar effect would be produced if the star was not spherical, and in revolving exposed a disk of varying area. A great variation could not thus be produced without the revolving body assuming a condition of unstable equilibrium. For the application of these principles to Iapetus, see *Annals of Harvard College Observatory*, xi. 264. This theory may ex

plain the variations of stars of the fourth class. Another theory would account for the changes of light by an opaque body or satellite passing between the star and the observer. It will be the object of the following discussion to show how fully this explanation will account for the variations of stars of the fifth class. A modification of this theory would replace the single eclipsing body by a cloud of meteorites. Such a theory will account for almost anything by suitably modifying the distribution of the meteors. If we can show that all the effects may be explained by a single body, or what amounts to the same thing, a spherical cloud of meteors so dense as to be opaque, there seems to be no reason for assuming a cloud of another form. All that can be claimed for any theory is that it explains all the facts. If then the computed variations of light agree with the observations within the limits of errors of observation, that is all that can be asked, and the theory should be accepted as the most probable explanation until some new fact is discovered which it will not explain, or some new theory which agrees equally well with observation and appears to be less improbable. The diminution in light might be caused by the interposition of a body which was self-luminous, instead of dark. We should then have a close double-star, one component of which passed in front of the other. If the orbit was circular, we should have two minima during each revolution, and at these times the star would appear of unequal brightness, unless the intrinsic brightness of the two bodies was the same. When the darker body passed in front of the brighter, the light would be less than when the brighter passed in front of the darker. If the orbit was elliptical there might be only one minimum. In the case of Algol more than half the light is cut off at the minimum; consequently one body must be darker than the other. As no second minimum has ever been observed, it is probable that the eclipsing body is not self-luminous.

We must now show that neither of the other theories named above will explain the variations of Algol and of other stars of the fifth class. The regularity of the variation disposes of the theory of a volcanic eruption, a collision, or a system of sun-spots. These effects also could scarcely be repeated so frequently without exhausting the source of energy from which they were derived. The theory that the variation is due to the revolution of the star appears more probable, and the regularity and shortness of the period add weight to it. On the other hand, it is difficult to account by this theory for the sudden changes in the light. If the light was reduced by a dark portion of the star being turned towards the observer, the minimum should last until, by the

revolution of the star, this part had been turned around so as to disappear on the other side. The short minimum observed could only be caused, according to this theory, by supposing a large dark star with a small bright spot near its polar regions, and that the pole was directed at such an angle from the observer that a large part of the spot would disappear for a short time during each revolution. Even then we have still the apparently insurmountable difficulty that the bright spot would change its apparent size and the angle at which it emitted its light to the observer, and therefore vary in brightness during the whole period of revolution. No such variation has been established in the light of Algol.

Before showing how far the theory of an eclipsing body will account for the observed phenomena, we must see what knowledge we have of these variations in light.

Only five stars are at present known to belong to the Algol class of variables. These are β *Persei*, S *Cancri*, λ *Tauri*, δ *Libræ*, and U *Coronæ*. Of these, the first is the only one whose variations are known with sufficient precision to justify a discussion in the present article. The variations of β *Persei*, or Algol, have been carefully studied by three observers, Argelander, Schmidt, and Schönfeld. Argelander's observations extend from 1840 to 1866, and are nearly two thousand in number. He compared Algol from time to time with the adjacent stars of nearly equal brightness, and noted the apparent difference in steps or grades (*Stufen*). Arranging his comparison stars in the order of brightness, and determining the number of grades between each from all his measures, he was then able to denote them all in grades. Thus, suppose at a given time he observed that Algol was slightly, if at all, brighter than star *A*, or that the difference was one grade; again, that it was perceptibly fainter than *B*, or differed from it by two grades; if then he found in his final discussion that $A = 12.5$ grades and $B = 14.9$, the first observation would make Algol 13.5 grades, and the second 12.9. These comparisons are all given in the *Bonn Observations*, vii. 315, but, unfortunately, they have not been reduced, so that at present no use can be made of them. I undertook their reduction, but was informed that this had been done at Bonn. No answer has, however, been received to letters of inquiry on this point.

The observations of Dr. Schmidt extend from 1846 to the present time, and the results up to 1875 are published in the *Astronomische Nachrichten*, lxxxvii. 193. His object was only to determine the time of the minima, and accordingly these only are given, without the comparisons. He also generally used a single comparison star, which

has the advantage of eliminating an error in estimating its brightness, but does not give a good determination of the light curve. Dr. Schönfeld observed Algol, according to the method of Argelander, from 1859 to the present time, and has given the results up to 1870, in the *Sechsunddreissigster Jahresbericht des Mannheimer Vereins für Naturkunde*, p. 70. He has not published his comparisons, but has given his resulting light curves, which will be made the basis of the following discussion. We must first reduce his grades to absolute measures, which is done in Table VII. The successive columns give the name of the comparison stars, the light in grades adopted by Schönfeld, the logarithm of the light as measured by Seidel (*Resultate photometrischer Messungen*, München, 1862), the logarithms of the light as measured by Wolff (*Photometrische Beobachtungen an Fixsternen*, Leipzig, 1877), after subtracting 0.232 to eliminate the constant difference between his measures and those of Seidel. The next column gives the difference between the measures of Seidel and Wolff, and shows that on the average they differ only .040, or a tenth of a magnitude. If L denotes the logarithm of the light, and g the corresponding number of grades, we may assume $L = a + g\,b$. This is only equivalent to admitting Fechner's law or assuming that Schönfeld's grades correspond to equal ratios of light. A solution by least squares gives $a = 8.446$ and $b = 0.025$, or $L = 8.446 + 0.025\,g$. The sixth column gives the value of L computed by this formula for the various values of g assumed by Schönfeld. The last two columns give the errors of Seidel and Wolff, assuming the estimates of Schönfeld to be exact. The average value of these differences is but little more than a tenth of a magnitude. Apparently, ι *Aurigæ* was estimated by Schönfeld about three tenths of a magnitude too bright, and δ *Persei* about two tenths too faint. Omitting these stars the errors would be reduced about one half.

TABLE VII. — Comparison Stars for β Persei.

Name.	Grades.	S	W − 0.232	W − 0.232 − S	Comp.	S − C	W − C
γ Androm.	23.4	9.038	9.021	—.017	9.031	+.007	—.010
ι Aurigæ	17.3	8.697	8.803	+.106	8.878	—.181	—.075
β Arietis	16.7	8.897	8.862	—.035	8.864	+.033	—.002
ε Persei	12.8	8.800	8.746	—.054	8.766	+.034	—.020
γ Persei	10.9	8.699	8.691	—.008	8.718	—.019	—.027
β Trianguli	9.1	8.716	8.716	.000	8.674	+.042	+.042
δ Persei	7.8	8.741	8.694	—.047	8.641	+.100	+.053
α Trianguli	3.5	8.531	8.588	+.057	8.534	—.003	+.054
ν Persei	0.9	—	8.435	—	8.468	—	—.033
				±.040		±.052	±.045

Schönfeld has given on page 84 of his memoir a table of his mean results, arranged in seventy-nine groups, seventy-two of them occurring within about four hours and a half of the minimum, and sixty-two within three hours of the minimum. He then drew an empirical curve through these points, and gives their residuals, which vary from +0.73 to —0.58 grades, and have an average value of 0.17 grades. Reducing this to logarithms, by multiplying by 0.025, gives 0.004, or only one hundredth of a magnitude. There are thirty-five changes of sign in the residuals, out of a possible seventy-one. There is, therefore, no reason to doubt that the curve represents the observations as nearly as possible.

The light in grades for intervals of every half-hour before and after the minimum is given in Table VIII. The successive columns give the time in hours, the corresponding light in grades before and after the minimum, the difference between these two, their mean, and the corresponding light expressed in logarithms. This is found by subtracting the light in grades from 20.8, which is assumed by Schönfeld as the full brightness of Algol, multiplying the result by 0.025 to reduce to logarithms, and taking the arithmetical complement. The number corresponding to this logarithm is given in the last column. It gives the light of Algol, its maximum light being assumed as 1.000.

TABLE VIII.—Light Curve of β Persei.

Hours.	Dec.	Inc.	D—I	$\frac{D+I}{2}$	Log L	Light.
0.0	5.56	5.56	0.00	5.56	9.619	0.416
0.5	6.26	6.20	+0.06	6.23	9.636	0.433
1.0	8.48	7.60	+0.88	8.04	9.681	0.480
1.5	12.05	9.81	+2.24	10.93	9.753	0.566
2.0	15.28	13.17	+2.11	14.22	9.836	0.685
2.5	17.35	15.78	+1.57	16.06	9.882	0.762
3.0	18.68	17.71	+0.97	18.20	9.935	0.861
3.5	19.59	19.19	+0.40	19.39	9.964	0.920
4.0	20.24	20.23	+0.01	20.24	9.986	0.968
4.5	20.70	20.75	—0.05	20.72	9.998	0.995
4.6	20.8	20.8	0.00	20.80	0.000	1.000

From this table it appears that the law respecting the increase of light is not the same as that of its diminution. At a given interval of time from the minimum, the light is greater when decreasing than when increasing. The mean value will first be considered, and the cause of this difference then discussed.

We shall first assume that the star and satellite present circular

disks, one uniformly bright, the other dark, and that the form of orbit is circular. Three cases may occur, corresponding to a total, an annular, and a partial eclipse of the star. In the first case, all the light would be cut off for a longer or shorter time; in the second, the minimum light would be maintained during the transit of the satellite across the face of the star; and in the third case the light would diminish until the minimum was attained and then immediately begin to increase. Algol appears to belong to the last of the classes. We must next determine the relative diameters of the satellite and star. A minimum diameter of the satellite may be computed from the minimum light. To reduce the light to 0.416, or to cut off 0.584 of the light, the diameter of the satellite must be at least $\sqrt{0.584} = 0.764$ times that of the star. In this case it would just pass completely on to the disk before it began to pass off. No maximum can be determined in this way, so that the diameter is only limited between 0.764 and infinity. A change in diameter will, however, produce a change in the law of variation of the light. We may deduce the diameter from the values agreeing most nearly with observation. We must now determine the amount of light remaining when the star is partially eclipsed by a satellite of radius r. The radius of the star is taken as the unit. The area of the segment of a circle of radius unity whose versed sine is z, is equal to $\text{versin}^{-1} z - (1 - z)\sqrt{2z - z^2}$. A table is given in the eighth edition of the *Encyclopædia Britannica*, xiv. 525, Art. *Mensuration*, which gives this quantity for values of z varying by hundreds from 0.00 to 1.00. The portion of the disk cut off will always be composed of two segments having the radii 1 and r, and having a common chord which may be computed when we know the distance of the centres. The area of each may be taken from the table, multiplied by the square of the radius of its circle, and the two areas added. This will give the required diminution in light.

If now we assume r the radius of the satellite, several of the elements may be computed.

The period of revolution of the satellite is given with much precision from the observations of the minima. It appears to undergo slight changes, but may be assumed for the present time to equal 2 days 20 hours 48.9 minutes. Calling w the longitude of the satellite in its orbit reckoned from its minimum, the mean change in w per hour will equal $5^\circ.023$. Since the beginning and ending of the obscuration precede and follow the minimum by $4^h\ 35^m$, the corresponding values of w will be $337^\circ.0$ and $23^\circ.0$. At these points the centre of the satellite will be at a distance $(1 + r)$ from the centre

of the star, or the disks will touch each other. They correspond to the first and last contacts of an eclipse. The orbit is projected into an ellipse whose major axis, a, equals the true distance of the centres, and whose minor axis, b, equals the distance at the time of greatest obscuration. When $r = 0.746$, $b = 1 - 0.746 = 0.254$. For other values of r, b must be determined from a computation of the area eclipsed, by successive approximations, until such a value is found as will reduce the light to 0.416. If x and y are the co-ordinates of the point in the orbit reached by the satellite at the time of first contact, by the properties of the ellipse $x = a \sin w$, and $y = b \cos w$. The square of the distance of the centres, or D^2, may be written

$$D^2 = (1 + r)^2 = (x^2 + y^2) = a^2 \sin^2 w + b^2 \cos^2 w$$
$$= a^2 - (a^2 - b^2) \cos^2 w.$$

Since $w = 23^\circ.0$,

$$(1 + r)^2 = 0.153\ a^2 + 0.847\ b^2.$$

Substituting the proper values of r and b, a may be deduced. The cosine of the inclination, i, of the orbit will equal $\frac{b}{a}$. The three lines of Table IX. give the values of a, b, and i computed by these formulas for the minimum value of $r = 0.764$, for $r = 1.000$, and for $r = 2.000$. There is no maximum value of r, which may be indefinitely large. Let R be any large value of r, and let $a = R + A$, $b = R + B$, and $D = R + d$; substituting these values in the formula, $D^2 = a^2 \sin^2 w + b^2 \cos^2 w$, the terms containing R^2 cancel each other, and we have $2\ R\ d = 2\ RA \sin^2 w + 2\ RB \cos^2 w$, omitting the terms not containing R, since when R is very large they may be neglected. Dividing both sides by $2\ R$ gives $d = A \sin^2 w + B \cos^2 w$. When $w = 23^\circ$, d must equal 1, and when $w = 0^\circ$, B will equal -0.132, since the arc of the large circle becomes sensibly a straight line, and the segment whose versed sine is $1.000 - 0.132$ has an area of 0.416, or the minimum area of the uneclipsed portion. From these values, we may deduce $A = 7.300$. The two axes, therefore, become $R - 0.132$ and $R + 7.300$. The inclination in this case continually diminishes as R increases, and would equal zero if R became infinite.

The residuals which will be deduced below at first led to the belief that the phenomenon might be that of an annular eclipse. This case has therefore been included to show the change effected in the variation of the light, although the residuals are not materially reduced. If the eclipse is annular, the value of r must be 0.764.

The value of b cannot be determined directly, but must be deduced from the times of internal and external contact. The interval between the internal contacts is assumed to be 24 minutes, or that during which the satellite moves through 2° of longitude. In the equation $D^2 = a^2 \sin^2 w + b^2 \cos^2 w$, we have for $w = 1°$,

$$D = (1 - r) = 0.236,$$

and as before for $w = 23°$,

$$D = 1 + r = 1.764.$$

From these conditions the values of a and b given in the last column of Table IX. are deduced.

TABLE IX. — ELEMENTS OF ORBITS.

Elements.	$r = 0.764$	$r = 1.000$	$r = 2.000$	$r = R$	Ann.
Minor semi-axis, b	0.236	0.666	1.783	$R - 0.132$	0.223
Major semi-axis, a	4.480	4.872	6.427	$R + 7.300$	4.482
Inclination, i	87°.0	82°.1	73°.9	Small.	87°.1

We must next compute the amount of obscuration at the end of each half-hour, for the various values of r. The distance between the centres is first computed by the equation $D^2 = a^2 - (a^2 - b^2) \cos^2 w$, substituting successively, w = 2°.5, 5°.0, 7°.5, 10.°0, 12°.6, 15°.1, 17°.6, and 20°.1. The first part of Table X. gives the values of D corresponding to those assigned to r at the head of each column. The triangles formed by the centres of the two bodies and one end of the segment now become known, since their three sides equal 1, r, and D. Calling the angle at the centre of the luminous body α, we have $r^2 = 1^2 + D^2 - 2\,D \cos \alpha$. From this we deduce $\cos \alpha$ and versin α, or the height of the segment bounded by the circle having a radius unity. The height of the other segment will equal $R - D + \cos \alpha$, from which the areas of the segment, and consequently of the uneclipsed portion, may be deduced. This area is given in the second portion of the table. For comparison the observed light is repeated in the last column from the last column of Table VIII. The residuals, or the observed values minus those computed with each value of r, are given in the third part of Table X. The residuals are all zero when the time equals 0.0 or 4.6, and are therefore omitted. The average residuals are given in the last line.

TABLE X. — DISTANCES OF CENTRES.

Hours.	0.764	1.00	2.00	$R-D$	0.764
0.0	0.236	0.666	1.783	—0.132	0.223
0.5	0.307	0.700	1.806	—0.116	0.298
1.0	0.463	0.794	1.865	—0.072	0.457
1.5	0.629	0.917	1.957	—0.005	0.625
2.0	0.811	1.072	2.081	+0.092	0.807
2.5	0.988	1.233	2.223	+0.211	0.986
3.0	1.191	1.425	2.404	+0.375	1.190
3.5	1.373	1.606	2.583	+0.548	1.372
4.0	1.556	1.789	2.773	+0.748	1.555
4.6	1.764	2.000	3.000	+1.000	1.764

LIGHT OF UNECLIPSED PORTION.

Hours.	0.764	1.00	2.00	R	0.764	Obs.
0.0	0.416	0.416	0.416	0.416	0.416	0.416
0.5	0.434	0.436	0.432	0.427	0.430	0.433
1.0	0.500	0.491	0.469	0.454	0.497	0.480
1.5	0.579	0.562	0.527	0.497	0.578	0.566
2.0	0.668	0.648	0.603	0.559	0.667	0.685
2.5	0.751	0.731	0.686	0.633	0.750	0.762
3.0	0.838	0.822	0.785	0.733	0.838	0.861
3.5	0.907	0.898	0.874	0.831	0.907	0.920
4.0	0.968	0.959	0.949	0.927	0.968	0.968
4.6	1.000	1.000	1.000	1.000	1.000	1.000

RESIDUALS.

Hours.	0.764	1.00	2.00	R	0.764
0.5	— .001	— .003	+ .001	+ .006	+ .003
1.0	— .020	— .011	+ .011	+ .026	— .017
1.5	— .013	+ .004	+ .039	+ .069	— .012
2.0	+ .017	+ .037	+ .082	+ .126	+ .018
2.5	+ .011	+ .031	+ .076	+ .129	+ .012
3.0	+ .023	+ .039	+ .076	+ .128	+ .023
3.5	+ .013	+ .022	+ .046	+ .089	+ .013
4.0	.000	+ .009	+ .019	+ .041	.000
	± .012	± .020	± .044	± .077	± .012

The residuals are all expressed in terms of the full light of the star. They therefore represent a larger error expressed in logarithms, or stellar magnitudes, when the star is faint than when it is bright. If reduced to logarithms their mean values become .008, .012, .027, .049, .008. Dividing these quantities by 0.4 to reduce them to magnitudes,

we see that while a large value of r would give an average residual of over one tenth of a magnitude, the value of $r = 0.764$ would make this quantity less than two hundredths of a magnitude. In all of them, however, there is a distinct systematic variation, the computed light being too small when t is large, and sometimes becoming too large when t is small. It appeared that this error might be reduced by assuming that the eclipse was annular, or that the light retained its minimum value for a short time. The corresponding residuals are given in the last column. They reduce the positive residuals when the star is faint, but do not sensibly affect the others, although the time between the internal contacts is assumed to be twenty-four minutes. The observations scarcely admit so great an interval, and certainly would not justify its increase. As the average residual is not diminished by the assumption of an annular eclipse, and as the observations do not indicate that the light remains constant during the minimum, we cannot do better than to assume the value of $r = 0.764$, and adopt the values of the second column of the table.

Several explanations may be offered of the small systematic error that remains. The most plausible seems to be that derived from the residuals given in the last column of Table VII. They show that, from a comparison of the estimated grades of Schönfeld with the measures of Wolff, that Schönfeld estimated the light too faint when the star was faint, and too bright when the star was bright. In other words, that a grade did not have the same values when expressed in logarithms for a faint as for a bright star. Assuming the photometric measures of Wolff to be free from systematic error, we should therefore increase the estimates of Schönfeld when the star was faint, and diminish them when it was bright, without affecting the actual maximum and minimum values. Such a correction would make the systematic error noted above disappear, or even give it an opposite sign. This view receives a slight confirmation from the measures of Seidel, but the accidental discrepancies far exceed this small systematic error. We may therefore conclude that the computed light agrees with observation as closely as the brightness of the fundamental stars is at present known, and there is no evidence of a real systematic difference between the two.

Another explanation of the residuals in Table X. has suggested itself. The presence of lines in stellar spectra leads to the belief that the stars, like our Sun, are surrounded by an absorbing atmosphere. They also, therefore, probably resemble it in presenting a disk brighter in the centre than at the edges, owing to the greater thickness of the

atmosphere and consequent greater absorption at the edges. The effect of such an absorption is best determined by the consideration that if, owing to absorption, the average light of the eclipsed portion is less than that of the whole disk, the effect of the atmosphere will be to diminish the proportion of the light cut off; in the opposite case, it will increase it. Now when a small portion only of the star is eclipsed, evidently the average light of this portion, since it lies near the edge, must be less than that of the whole. The atmosphere, although then diminishing the light of the remaining portion, will not reduce it as much as it does that of the entire disk; the relative light will therefore be increased. On the other hand, when a large part of the eclipsed portion is from the central and brightest portion the opposite effect will be produced. We should therefore expect, when t is large, that the computed light should be increased. When t is small, it may be diminished. In the case of the Sun the effect is so slight, except close to the borders, that the previous explanation seems more probable.

We return now to the consideration of differences in the rate of diminution and increase of the light. The observations ought to give this quantity with much accuracy. An error in estimating the light of the standard stars will not sensibly affect it, since the same stars are used in measuring the increase and diminution. The effect of atmospheric absorption is reduced, since some of the comparison stars are always above and others below the variable, and besides, although, when observed before passing the meridian, the star is brighter when increasing than when diminishing, yet the opposite effect is produced when the star is west of the meridian. Nevertheless this difference is doubted by many astronomers, and if it exists it is evident that an important correction should be applied to the observed minima of Algol. If the curve found by Schönfeld is correct, an error of ten minutes in the time of the minimum might be caused by comparing with a star like ε *Persei*, having a brightness of about twelve grades, and taking the mean of the times when the two stars appeared equal.

Three explanations may be offered for this phenomena. First, that the satellite is not spherical, but egg-shaped, and that the large end is turned forwards; or that the satellite is of unequal density, and that the heaviest portion is forward. In this case the centre of gravity of the disk would follow that of the satellite, or for a given distance of the centres the interposed area would be greater when the satellite was passing off, than when coming on. So great a deviation from the spherical shape would be needed to produce the observed difference

that this theory does not seem very probable. We should also, in this case, assume that the time of revolution was exactly equal to that of rotation of the satellite. A second explanation would assume that one portion of the disk of Algol was darker than the rest, so that when the satellite entered the disk it would cut off the dark portion, or affect the light less than when passing off and obscuring the brighter parts. In this case we must assume that Algol does not rotate, or it would show a variation independent of the eclipse by its satellite. Its axis of rotation might be parallel to the path of the satellite and the variations in light on its surface be distributed in zones, but such a theory seems improbable. The third explanation is that the orbit of the satellite is elliptical, and that the difference is due to the varying velocity of the satellite.

An analytical solution of this problem may be found by reducing the observed light to distances of centres, either by interpolation from the values computed above, or by successive approximations. The case then becomes that of a binary star, in which we have given the period and a number of distances, but no position angles. It is of course impossible to deduce the position angles of the peri-astron or other point of the orbit, but its other dimensions may be determined. The solution of this problem will be undertaken at another time should the accumulation of observations of Algol and other similar stars render it desirable. For the present, it will be sufficient to obtain an approximate solution. The nature of the variation is not so simple as would appear at first sight ; since the observed time of increase equals that of diminution, we must assume that the apparent motion, when compared with that in a circular orbit, is less at the beginning and end, and greater in the middle of its path. The satellite must therefore either pass its peri-astron during the eclipse, or it must be approaching this point, so that the increased obliquity of its path to the line of sight will produce the apparent diminution in its motion. An ellipse was constructed, having an eccentricity of 0.5 and divided into thirty-two parts, corresponding to the position of the satellite at the end of each thirty-second of its time of revolution. The eccentric anomaly was derived from the mean anomaly by the tables of Dr. Doberck, *Astronomische Nachrichten*, cxii. 275.

As the time of eclipse is very nearly one eighth of that of revolution, four of these divisions correspond to the passage of the satellite over the star. Laying this ellipse on a sheet of rectangular paper and turning it around its focus, the effect of a change in the position of the peri-astron could be determined. The problem is greatly sim-

plified by the fact that the apparent path of the satellite during the eclipse is nearly rectilinear. It was found that, if the longitude of the line of nodes was made equal to 17°, the periods of ingress and egress would be nearly equal. The peri-astron then happens to coincide with the point of egress. The variation in light due to this orbit is compared with observation in Table XI. The successive columns give the time, the observed light in grades, the logarithm of this light, and its value compared with the full light of the star. The next column gives the light already found in the second column of the second part of Table X., and which may be called *A*. The next column gives the variation in light for the elliptical orbit assumed above, which will be denoted as *B*. The second part of the table gives the residuals found by subtracting these values of the light from those observed. The last columns give the residuals found by subtracting the logarithms of these quantities. Although the residuals even of *A* are not very large, they are systematic, being positive when the light diminishes, and negative when it increases. The residuals *B* are much smaller than those of *A* during ingress, but they are larger during egress. In other words, while the systematic error of ingress has been nearly eliminated, a nearly equal error has been introduced during egress. Accordingly the average residual is not diminished. We have so far adopted the times of first and last contact given by Schönfeld. An inspection of the table from which he derived his curve shows that the weight he assigns to his observations when more than three hours from the minimum is small, and that consequently the times of contact must be somewhat uncertain. The exact time of minimum must also be uncertain, although to a less degree than that of the two points just mentioned. An approximate solution by least squares was therefore made, with the times of contact and of minimum as unknown quantities. One half weight was given to the equations of condition formed from the observed terms of contact. From this a correction to the observed minimum was found of 5 minutes, or the true minimum appears to occur nearly one tenth of an hour later than that given by the curve. The time of first contact should also be diminished by about 2 minutes, and the time of last contact increased by about 13 minutes. The columns *C* give the values of the light and of the residuals corresponding to this orbit. The third place of decimals is not always exact, as this would have involved a great increase in the labor of computation and the accuracy attained appears to be all that is at present justified by the observations.

The residuals thus obtained are quite satisfactory as regards their

magnitudes and the number of changes of sign, but the orbit is open to a criticism of a wholly different kind. Its semi-axis major is only 3.55, and as the eccentricity is 0.500, the distance of the centres at peri-astron is 1.775. Now as the radius of the star is 1.000 and of the satellite .764, it is evident that, although they would not actually

TABLE XI. — COMPARISON OF ORBITS.

Hours.	Grades.	Log *L*	*L*	*A*	*B*	*C*	*D*
4.6	20.8	1.000	1.000	1.000	0.999	1.000	1.000
4.0	20.24	0.986	0.968	0.968	0.982	0.986	0.987
3.5	19.59	0.970	0.933	0.907	0.945	0.949	0.937
3.0	18.68	0.947	0.885	0.838	0.883	0.890	0.866
2.5	17.35	0.914	0.820	0.751	0.809	0.815	0.788
2.0	15.28	0.862	0.728	0.668	0.714	0.725	0.697
1.5	12.05	0.781	0.604	0.579	0.613	0.626	0.601
1.0	8.48	0.692	0.492	0.500	0.517	0.534	0.518
0.5	6.26	0.636	0.432	0.434	0.440	0.450	0.446
0.0	5.56	0.619	0.416	0.416	0.416	0.416	0.416
0.5	6.20	0.635	0.432	0.434	0.440	0.429	0.426
1.0	7.60	0.670	0.468	0.500	0.515	0.494	0.486
1.5	9.81	0.725	0.531	0.579	0.603	0.576	0.570
2.0	13.17	0.809	0.644	0.668	0.695	0.665	0.660
2.5	15.78	0.874	0.748	0.751	0.785	0.754	0.753
3.0	17.71	0.923	0.838	0.838	0.858	0.830	0.839
3.5	19.19	0.960	0.912	0.907	0.926	0.899	0.909
4.0	20.23	0.986	0.968	0.968	0.975	0.953	0.970
4.6	20.8	1.000	1.000	1.000	1.000	0.995	1.000

Hours.	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
4.6	.000	—.001	.000	.000	.000	.000	.000	.000
4.0	.000	—.014	—.018	—.019	.000	—.006	—.008	—.008
3.5	+.026	—.012	—.016	—.004	+.012	—.005	—.007	—.002
3.0	+.047	+.002	—.005	+.019	+.024	+.001	—.002	+.009
2.5	+.069	+.011	+.005	+.032	+.038	+.006	+.003	+.018
2.0	+.060	+.014	+.003	+.031	+.037	+.008	+.002	+.019
1.5	+.025	—.009	—.022	+.003	—.018	—.007	—.016	+.002
1.0	—.008	—.025	—.042	—.026	—.007	—.022	—.036	—.022
0.5	—.002	—.008	—.018	—.014	—.002	—.008	—.017	—.013
0.0	.000	.000	.000	.000	.000	.000	.000	.000
0.5	—.002	—.008	+.003	+.006	—.003	—.009	+.003	+.006
1.0	—.032	—.047	—.026	—.018	—.029	—.042	—.024	—.017
1.5	—.048	—.072	—.045	—.039	—.038	—.055	—.035	—.031
2.0	—.024	—.051	—.021	—.016	—.016	—.033	—.014	—.011
2.5	—.003	—.037	—.006	—.005	—.002	—.021	—.003	—.003
3.0	.000	—.020	+.008	—.001	.000	—.011	+.004	—.001
3.5	+.005	—.014	+.013	+.003	+.002	—.007	+.006	+.001
4.0	.000	—.007	+.015	—.002	.000	—.003	+.007	—.001
4.6	.000	.000	—.005	.000	.000	.000	—.002	.000
	±.018	±.019	±.014	±.012	±.012	±.013	±.010	±.009

touch, yet they would come so near that the least disturbance would at once produce a catastrophe. This, therefore, gives the limiting value to the eccentricity. A computation with a smaller eccentricity gave less satisfactory residuals. The question now arises, will it not be possible to satisfy the observations by returning to the circular elements, since we have permitted a change in the times of contact and of minimum. Columns *D* give the residuals for a circular orbit with a diminution of 0.1 hour in the time of minimum, and assuming that the periods of ingress and egress are each equal to 4.45 hours instead of 4.6 hours. In other words the ingress occurs about fourteen minutes later, and the egress two minutes earlier, than was assumed by Schönfeld.

The errors which remain, even in the last orbit, are not wholly accidental; but their values are so small, and the changes of sign so frequent, that it is not safe to base important conclusions upon them. Their average value is only .012, or expressed in logarithms .009, and in magnitudes .02. Accordingly, we may compute the variation in the light of Algol, which shall not differ from observation on an average more than a fiftieth of a magnitude. If then this is not the true cause of the variation of the light, it at least satisfies it well within the errors of observation. The orbit *D* may therefore be adopted as representing the law of variation as well as it is at present known.

The stellar magnitude of Algol is about 2.0, so that by Table II., if its brightness equals that of the Sun, its diameter will equal $0''.006$. The diameter of the orbit of the satellite will be about $0''.028$. The motion of the bright star, if its density is the same as that of its satellite, will equal $0''.009$, since its mass in this case will be to that of its satellite as 1.000 is to 0.446. It would therefore be useless to attempt to observe the motion micrometrically. For the same reason, there seems to be no means by which we can determine the position angle of the satellite, or the direction of the axes of the ellipse into which the orbit is projected. Even if future observations should render a larger value of the radius probable, the motion would be scarcely perceptible micrometrically. If $r = 2.000$, the diameter of the orbit becomes $0''.08$ and the motion of Algol about $0''.07$. It would be difficult to measure so small a quantity, although, as it is traversed in less than a day and a half, many sources of systematic error would be eliminated.

Below are given, in successive columns, the corresponding values of several elements of the orbits *A*, *B*, *C*, and *D*. The diameter of Algol is assumed to be $0''.006$. The times are given in minutes from the

minimum adopted by Schönfeld. A negative sign denotes that the time precedes the minimum; a positive, that it follows it.

Elements.	*A.*	*B.*	*C.*	*D.*
Eccentricity	0.00	0.50	0.50	0.00
Semi-axis major	0″.0134	0″.0109	0.″0106	0″.0138
Inclination	87°.0	84°.3	84°.2	87°.1
Longitude of nodes	—	17°	17°	—
Time of first contact	—276	—279	—278	—261
" minimum	0	— 1	+ 5	+ 6
" last contact	+276	+273	+289	+273

The elliptical orbits B and C are much smaller than the others. Since the eclipse takes place near the peri-astron, the angular motion is so great that the radius vector must be reduced to maintain the same duration of eclipse.

To give a more tangible idea of the dimensions of this system a projection is given of the orbit denoted by D in its own plane in Fig. 1,

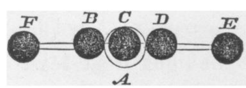

Fig. 1.

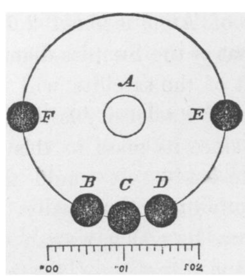

Fig. 2.

and as seen from the Earth in Fig. 2. In both projections A denotes the primary, B the satellite at first contact, C when half across the disk, D at the last contact, and E and F at its elongations. The scale is one hundredth of a second to a centimeter. Accordingly, if Fig. 2 is removed to a distance of 206 kilometers, or about 120 miles, it would appear of the same size as Algol when seen from the Earth.

The application of the spectroscope to this binary star offers a most interesting field for work. Assuming the same data as before, we find the circumference of the orbit equals $2\,\pi \times 0''.0138 = 0''.087$; or

multiplying by 0.446 and dividing by 1.446 will give a motion of Algol of $0''.027$ in each revolution. This corresponds to $3''.43$ annually. If the parallax of Algol is $0''.1$, this would correspond to a velocity of about 160 kilometers (100 miles) per second. Substituting the values in the equation on page 9, $v = 13\ l$, we have $l = 2.6$, or the F line would be deviated through an interval equal to nearly half the space between the D lines. Moreover, as this quantity would be alternately positive and negative every thirty-five hours, the systematic errors which are so troublesome in such measures could be eliminated, and the quantity to be observed would be doubled. If the parallax of the star is more than $0''.1$ the motion would be less, but on the other hand the parallax would then become a suitable object for micrometric measurement. If the parallax is much less than $0''.1$ the motion would be so large that its variations might be determined with some accuracy, and the form of the orbit computed from the varying velocity along the line of sight. These measures would also determine the dimensions of the orbit, and if we assume the value of the brightness, l, they would give the distance and parallax of the star. The spectrum of Algol has already been examined without the detection in it of any peculiarity. The time selected for observation would be more likely to be near its minimum, to detect any changes in the spectrum accompanying its variation in light. But this is the very time when the motion along the line of sight is zero, which may be the reason why this phenomenon has as yet escaped detection.

Two objections have been offered to the theory that the variation in light was due to the interposition of a non-luminous satellite. First, the large size of the satellite; and, secondly, the rapidity of its motion. It has been said that, according to the prevalent theories regarding the formation of the stars, so large a body could not well have lost all its heat while the luminous star is still so bright. This argument would have some force, if we were sure of the true origin of the stars, and also if we knew that both bodies are of the same age. They may, however, have had a wholly independent origin, and have come together through their proper motion, under the influence of a resisting medium or other disturbing force.

The objection to the rapidity of motion cannot be defended in that form. By the law of gravitation we can compute what should be the velocity with a given density, and the only proper criticism would be that to produce the observed velocity an improbable density would be required. To determine this density we may use the formula of

page 6 for the equivalent diameter of the system, using as a unit the radius of the star. We thus find, $b = 0.00933\, a P^{-\frac{2}{3}} = 0.00933 \times 4.60\, (0.00785)^{-\frac{2}{3}} = 1.087$. Accordingly, a body having the density of the Sun, and a diameter but little more than half that of Algol, would give the observed time of revolution to the satellite. If, therefore, the velocity is remarkable, it is remarkable that it is not greater. If the satellite of Algol has a diameter of 0.764, and its density equals that of the primary, its relative mass will be 0.446. The two bodies combined would form a sphere having a radius of 1.130 and a diameter of 2.260. This is 2.08 times that of the equivalent diameter, and shows that the average density can be only 0.11 of that of the Sun, or about one seventh of that of water.

It may be noted that the density affords a means of distinguishing between a satellite and a spherical cloud of meteors. If the individual meteors were very minute, they might completely cut off the light, and yet bear a very small ratio in volume to the space between them. Accordingly, if the density of the eclipsing body could be shown to be very small, we might infer that it was composed of meteorites. In this case the motion of Algol would be insensible, as seen in the spectroscope.

The observed times of minima of Algol seem to show that its period has undergone a diminution during the last century. Such a change is easily explained on the theory of a secondary satellite. The disturbance caused by a third body, or by a resisting medium, might very sensibly vary the period from year to year. The law of this change is not yet known, but its nature is shown in Table XII. The minima are distinguished by successive numbers, E, that occurring on Jan. 1, 1800, being designated as 0. Those preceding 9000 have been arranged in groups of 500 each. Since 1870 the observations of each year are grouped together. The successive columns of the table give a current number, the mean of the numbers of the minima, the corresponding year and tenth and the number of minima included in the group. In the last nine groups, which relate to a single year, the minimum corresponding to opposition is used, instead of the mean of those observed.

The first eleven sets were observed by various astronomers; sets 12 to 18 were made by Argelander; sets 19 to 21 by Schönfeld; and sets 22 to 30, by Schönfeld and Schmidt. Sets 18 and 19 relate to the same period of 500 revolutions from 7500 to 8000. The fifth column is found by subtracting from the observed time that given by the formula of Schönfeld on page 94 of his memoir, —

Ep. $E =$ 1867, Jan. $\overset{\text{d.}}{0}\ \overset{\text{h.}}{11}\ \overset{\text{m.}}{1.2}$ M. Z. Paris $+$ $\overset{\text{d.}}{2}\ \overset{\text{h.}}{20}\ \overset{\text{m.}}{48.9}$ (E — 8534).

For the earlier observations the reduction given by Argelander (*Bonn Observations*, p. 347) are used, after reducing them to the above formula by subtracting $355^{\text{m.}} - 0.0749\ E$. The sixth column gives the ordinates of a smooth curve without points of inflection drawn through the points whose abscissas and ordinates are respectively given in the third and fifth columns. The last column gives the difference between the fifth and sixth columns.

TABLE XII. — Minima of Algol.

No.	Mean Epoch.	Date.	No. Min.	Obs.	Curve.	O. — C.
1	— 2101	1783.5	27	— 510	— 507	— 3
2	— 1860	1785.4	17	— 488	— 489	+ 1
3	— 1308	1789.8	17	— 450	— 446	— 4
4	— 706	1794.5	6	— 400	— 401	+ 1
5	— 250	1798.0	10	— 368	— 367	— 1
6	+ 214	1801.7	2	— 308	— 332	+ 24
7	+ 734	1805.7	2	— 280	— 293	+ 13
8	+ 1831	1814.4	2	— 211	— 210	— 1
9	+ 2282	1817.9	6	— 180	— 177	— 3
10	+ 2574	1820.2	5	— 155	— 155	0
11	+ 3212	1825.2	3	— 114	— 108	— 6
12	+ 4081	1832.0	2	— 44	— 46	+ 2
13	+ 5259	1841.3	16	+ 25	+ 24	+ 1
14	+ 5741	1845.1	4	+ 37	+ 25	+ 12
15	+ 6154	1848.4	6	+ 24	+ 24	0
16	+ 6838	1853.7	16	+ 21	+ 20	+ 1
17	+ 7308	1857.4	17	+ 10	+ 15	— 5
18	+ 7688	1860.4	4	— 1	+ 11	— 12
19	+ 7799	1861.3	5	— 5	+ 9	— 14
20	+ 8374	1865.9	12	+ 2	+ 2	0
21	+ 8791	1869.0	15	— 1	— 5	+ 4
22	+ 9026	1870.9	13	+ 1	— 8	+ 9
23	+ 9153	1871.9	12	— 4	— 10	+ 6
24	+ 9281	1872.9	19	— 7	— 12	+ 5
25	+ 9408	1873.9	16	— 1	— 14	+ 13
26	+ 9535	1874.9	4	— 8	— 17	+ 9
27	+ 9662	1875.9	9	— 7	— 20	+ 13
28	+ 9789	1876.9	13	— 22	— 23	+ 1
29	+ 9916	1877.9	9	— 46	— 26	— 20
30	+10043	1878.9	4	— 29	— 29	0

The numbers in the last column nearly equal the accidental errors of observation. There is a slight grouping of negative signs about 1860, and of positive signs soon after 1870. This could not be avoided without giving to the curve a point of inflection. The average value of these residuals is about six minutes, which shows the accordance to be expected from any assumed formula.

Adopting the curve described above as representing the true variation, its ordinates for every ten years have been read off, and are given in the third column of Table XIII. The direction of its tangent has also been determined, and the seconds of the resulting period is entered in the fourth column. To this is to be added $2^{d.}\ 20^{h.}\ 48^{m.}$ The second column gives approximately the corresponding value of *E*.

TABLE XIII. — VARIATION OF PERIOD.

Year.	*E*.	Curve.	Period.
		m.	s.
1780	— 2545	— 541	58.6
1790	— 1273	— 444	58.6
1800	0	— 348	58.5
1810	+ 1273	— 252	58.5
1820	+ 2545	— 157	58.4
1830	+ 3818	— 64	58.3
1840	+ 5090	+ 16	56.6
1850	+ 6363	+ 24	53.7
1860	+ 7635	+ 11	53.3
1870	+ 8908	+ 7	53.0
1880	+10180	+ 31	52.7

An inspection of the curve of variation of the times of minimum shows that a curious change took place between 1830 and 1850. Before then, the period given by Wurm of $2^{d.}\ 20^{h.}\ 48^{m.}\ 58.5^{s.}$ represents the observations well; after 1850, the formula of Schönfeld appears to be more nearly correct. There seems, during this interval, to have been a change of four or five seconds in the period, and that besides this there has been a small but gradually increasing diminution in the period throughout the century.

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